HIGH STRAIN RATE MATERIAL TESTING

Split Hopkinson Pressure Bar

SHPB Equations used in SURE-Pulse
SURE-Pulse® Calculations: Descriptions & Examples

SURE-pulse uses several calculations in reaching its results.

**Young’s modulus of bar**

The Young’s modulus of the bar is found from the wave speed and the density.

The wave speed is found in the bar calibration by using the distance between the strain gauges and the time between pulses.

$$\text{Wave speed} = \frac{\text{Distance between strain gauges}}{\text{Time Between Pulses}}$$

For example, if the pulse travels a distance of 1 m in 201 $\mu$s in a steel bar, the wave speed can be found as follows.

$$\text{Wave speed} = \frac{1 \text{ m}}{201.2 \mu s} = 4970 \frac{\text{m}}{\text{s}}$$

The Young’s modulus is found from the wave speed and density.

$$\text{Young's Modulus} = \text{Wave Speed}^2 \times \text{Density}$$

For example, if the same steel bar that was used above has a density of 8100 $\frac{kg}{m^3}$, the Young’s modulus can be found as follows.

$$E = \left(4970 \frac{m}{s}\right)^2 \times 8100 \frac{kg}{m^3} = 200 \text{ GPa}$$

**Strain calculation in bar**

The strain gauge readings in volts need to be converted into strain. This requires the resistance of the strain gauge, the resistance of a calibrating resistor, the gauge factor of the strain gauge, and the voltage to which system is calibrated. (Vishay Micro-Measurements, 2002) Please read the Vishay manual for further details on strain calculation.

$$\text{Strain} = \frac{\text{Voltage reading} \times \text{Gauge Resistance}}{\text{Voltage Calibrated} \times \text{Gauge Factor} \times (\text{Gauge Resistance} + \text{Calibration Resistance})}$$

For example if the voltage reading on the strain gauge is 2V, the gauge resistance is 120 $\Omega$, the voltage the gauge was calibrated at was 2V, the gauge factor is 2, and the calibrating resistor has a resistance of 59940 $\Omega$ the strain in the bar can be found as follows.

$$\text{Strain} = 2V \times \frac{120 \Omega}{2V \times 2 \times (120 \Omega + 59940 \Omega)} = .001$$
Sample Stress and Strain Calculations

Force on the sample is equal to the transmitted force in the pulse transmission bar if the sample is in equilibrium. To find the stress in the sample, the ratio of areas of the bar and the sample is multiplied by the stress in the bar.

\[
\sigma_{\text{sample}}(t) = \frac{\text{Cross Sectional Area Of Bar}}{\text{Cross Sectional Area Of Sample}} E_T \epsilon_T(t)
\]

Where \( E_T \) is the elastic modulus of the transmission bar, and \( \epsilon_T(t) \) is the strain in the transmission bar at time \( t \).

For example, with the a 2 cm steel bar and a cylindrical sample with a diameter of .5 cm, for a strain of .001 in the transmission bar, the corresponding stress in the sample can be found as follows.

\[
\sigma_{\text{sample}}(t) = \frac{.00314 \, m^2}{.000196 \, m^2} \times 200 \, GPa \times .001 = 800 \, MPa
\]

The strain rate of the sample \( \dot{\epsilon}_{\text{sample}}(t) \) can be found using one dimensional wave theory and an assumption of equilibrium.

\[
\dot{\epsilon}_{\text{sample}}(t) = -2 \frac{\text{Wave Speed Incident Bar}}{\text{Length of Sample}} \epsilon_T(t)
\]

Where \( \epsilon_T(t) \) is the strain of the reflected pulse in the incident bar at time \( t \).

For example, with the above mentioned steel bar and a sample of 1 cm, for a strain of .001 in the reflected pulse in the incident bar, the corresponding strain in the sample can be found as follows.

\[
\dot{\epsilon}_1(t) = -2 \frac{4970 \, m}{.01 \, m} \times .001 = 994 \frac{1}{s}
\]

Sample strain can be easily found from strain rate using an integral.

\[
\epsilon_{\text{sample}}(t) = \int_{t_{\text{begin}}}^{t_{\text{end}}} \dot{\epsilon}_{\text{sample}}(t) \, dt
\]

Time is indexed from the beginning of the pulses, so the first strain gauge reading that is determined to be in each selected pulse is at \( t = 0 \) and is considered to occur at the same time in the sample.

If you assume that the above mentioned strain rate is constant over 6 \( \mu s \), the sample strain can be found as follows.

\[
\epsilon_1(t) = \int_{0}^{6 \, \mu s} 994 \frac{1}{s} \, dt = .05964
\]
Dynamic Force

The force on the back face can be found from the transmitted strain $\varepsilon_t(t)$

$$\text{Back Face Force} = \text{Area Of Bar} \times E_t\varepsilon_t(t) \quad \text{e.g.} \quad 0.003142 \, m^2 \times 200 \, GPa \times 0.001 = 62840 \, N$$

The force on the front face can be found from the difference in the strains of the incident $\varepsilon_i(t)$ and reflected $\varepsilon_R(t)$ pulses.

$$\text{Front Face Force} = \text{Area Of Bar} \times E_i\left(\varepsilon_i(t) - \varepsilon_R(t)\right)$$

For example, with a 2 cm steel bar with a Young’s modulus of 200 GPa, a strain of 0.001 on the incident pulse, and a strain of 0.0009 on the reflected pulse the force on the front face of the sample can be found as follows.

$$0.003142 \, m^2 \times 200 \, GPa \times (0.001 - 0.0009) = 6284 \, N$$

Noise Issues

Often the incident bar has a good deal of background noise due to the incident pulse having travelled through the bar. In order in minimize bias in the sample from noise, the voltage of the reflected pulse is adjusted down by the initial voltage.

$$\text{Voltage adjusted} = \text{Voltage Measured} - \text{Voltage baseline}$$

If noise is low (2.5% of signal or less), then the Voltage baseline $\approx 0$ and the result is no adjustment.

Energy Calculations

Bar alignment and shot quality can be evaluated using energy calculations. If the bars are closely aligned, the energy of the shot should stay level as it travels down the bar. Also, the energy from a non-trapped shot should closely match the energy from the striker bar.

There are two forms of energy in the bar, elastic and kinetic, and both are proportional to $\varepsilon_B^2$. More details on these numbers can be found in (Chen & Song, 2011) in the Conventional Bar section. The power travelling through the bar at any point in time can be found with the following equation.

$$\text{Power}(t) = \left(\frac{1}{2} \rho_B A_B C_B^3 + \frac{1}{2} A_B C_B E_B\right) \varepsilon_B^2(t)$$

For example, if you have a 2 cm steel bar that at some point in the pulse experiences a strain of 0.001, the power at that point can found as follows.

$$\left(\frac{1}{2} \times 8100 \, \frac{kg}{m^3} \times 0.003142 \, m^2 \times \left(4970 \, \frac{m}{s}\right)^3 + \frac{1}{2} \times 0.003142 \, m^2 \times 4970 \, \frac{m}{s} \times 200 \, GPa\right) \times (0.001)^2$$

$$= 312375 \, watts$$

Where $\rho_B$ is the density of the bar, $A_B$ is the area of the bar, $C_B$ is the wave speed of the bar, $E_B$ is the Young’s modulus of the bar, and $\varepsilon_B$ is the strain in the bar.
To find the energy of a pulse, integrate power over the whole pulse

\[ \text{Pulse Energy} = \int_{0}^{\text{end of pulse}} \text{Power}(t) \, dt \]

For example, a pulse in the steel bar with a strain of .001 over 60 \( \mu s \) can be found as follows using the power calculated above.

\[ \text{Pulse Energy} = \int_{0}^{60 \mu s} 312375 \text{ watts} \, dt = 18.74 \text{ joules} \]

**Number of Reflections**

The above equations assume that the sample is in equilibrium, it has been shown (Ravichandran \\& Subhash, 1994) that 4 reflections of the wave in the sample are necessary to reach equilibrium. The number of reflections in a sample are calculated as follows.

\[ \text{Number of Reflections} = \frac{1}{2} \frac{\text{Duration of Pulse}}{l_s/C_s} \]

\[ e.g. \text{Number of Reflections} = \frac{1}{2} \frac{60 \mu s}{.01 \text{ m}/6420 \text{ m/s (Aluminum)}} = 19.3 \]

Where the duration of the pulse is the duration of the transmitted pulse, \( l_s \) is the length of the sample, and \( C_s \) is the wave speed of the sample.

**References:**

